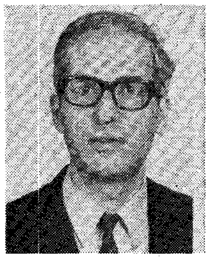

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Short Papers

Rectangular Waveguide with Two Double Ridges

D. DASGUPTA AND P. K. SAHA

Abstract—**An eigenvalue equation of a general structure having two arbitrary double ridges in a rectangular waveguide is derived. The cutoff wavelengths of two special cases with two symmetrically placed identical double ridges is computed numerically and their bandwidths are compared. The numerical solution of the eigenvector is also discussed and utilized in determining the gap impedance. As an example of the applications of such ridged waveguides, two varactor-tuned Gunn oscillators are briefly reported.**

I. Introduction

Recently, the authors presented an analysis based on Montgomery's work [1] for determining the eigenvalue spectrum of a rectangular waveguide with two symmetrically placed identical double ridges [2]. The numerical results indicated that such a waveguide would have adequate bandwidth for application in solid-state microwave oscillators. This structure can be generalized by considering two different double ridges at arbitrary locations in the waveguide. The structure treated in [2], [3] is then a special case of this general configuration. Another special case results when one of the two identical double ridges is inverted with respect to the other. The calculations show that the latter structure has a larger bandwidth compared to the former. In addition, some results on the calculation of the gap impedance are also presented. Following the analysis given in [2], we present only the final matrix eigenvalue equations without going into details.

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II. Ridged Waveguide

The generalized double-ridged waveguide structure is shown in Fig. 1(a). Two special cases, shown in Figs. 1(b) and 1(c), are referred to as "regular" and "inverted" structures, respectively.

III. Theory

A. Matrix Eigenvalue Equation

To solve the integral eigenvalue equation for TE modes by the Ritz–Galerkin technique, the transverse electric field at the kth aperture of the jth ridge ($j, k = 1,2$) is expanded as

$$E_{j,k}(y) = \sum_{i=0}^{N_{jk}} C_i^{(j,k)} \cos \frac{i\pi}{d_j}(y - h_j). \tag{1}$$

The resulting matrix equation for the eigenvalue k_c then takes the form

$$[H(k_c)]C = 0. \tag{2}$$

The vector C in (2) is given by

$$C = [C^{(1,1)T} C^{(1,2)T} C^{(2,1)T} C^{(2,2)T}]^T \tag{3}$$

where the superscript T denotes the transpose, $[H]$ is a matrix having the following partitioned form:

$$[H] = \begin{bmatrix} H_1 & H_2 & 0 & 0 \\ H_2 & H_3 & H_4 & 0 \\ 0 & H_5 & H_6 & H_7 \\ 0 & 0 & H_7 & H_8 \end{bmatrix}. \tag{4}$$

The eigenvalue equation is then

$$\det[H(k_c)] = 0. \tag{5}$$

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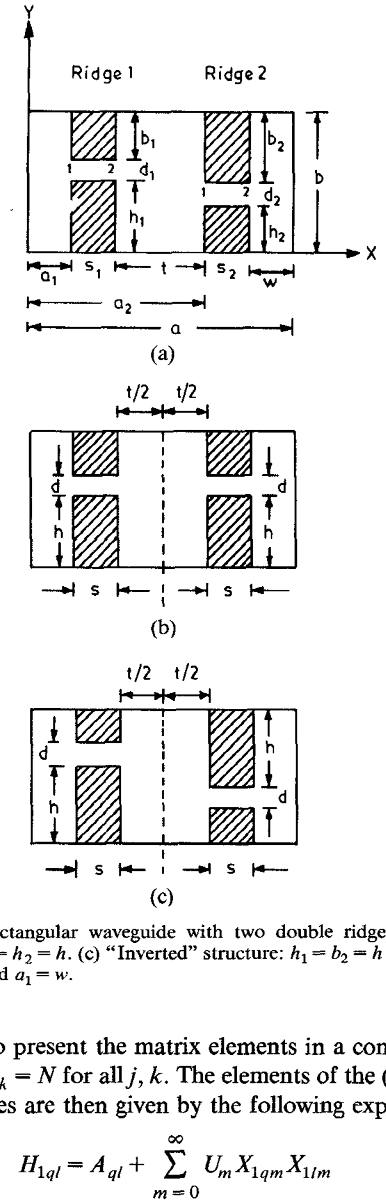

Fig. 1. (a) Rectangular waveguide with two double ridges. (b) "Regular" structure: $h_1 = h_2 = h$. (c) "Inverted" structure: $h_1 = b_2 = h$ with $d_1 = d_2 = d$, $s_1 = s_2 = s$ and $a_1 = w$.

In order to present the matrix elements in a compact form, let us assume $N_{jk} = N$ for all j, k. The elements of the $(N+1)\times(N+1)$ submatrices are then given by the following expressions:

$$H_{1ql} = A_{ql} + \sum_{m=0}^{\infty} U_m X_{1qm} X_{1lm} \tag{6a}$$

$$H_{3ql} = A_{ql} + \sum_{m=0}^{\infty} V_m X_{1qm} X_{1lm} \tag{6b}$$

$$H_{6ql} = B_{ql} + \sum_{m=0}^{\infty} V_m X_{2qm} X_{2lm} \tag{6c}$$

$$H_{8ql} = B_{ql} + \sum_{m=0}^{\infty} T_m X_{2qm} X_{2lm} \tag{6d}$$

$$H_{7ql} = -\delta_{ql}\epsilon_q d_2 \left(k_{x2q} \sin k_{x2q} s_2\right)^{-1} \tag{6e}$$

$$H_{5ql} = -\sum_{m=0}^{\infty} S_m X_{2qm} X_{1lm} \tag{6f}$$

where

$$A_{ql} = H_{2ql} \cos k_{x1q} s_1 \tag{7a}$$

$$B_{ql} = H_{7ql} \cos k_{x2q} s_2 \tag{7b}$$

$$U_m = g_m \cot k_{txm} q_1 \tag{7c}$$

$$V_m = q_m \cot k_{txm} t \tag{7d}$$

$$T_m = g_m \cot k_{txm} w \tag{7e}$$

$$S_m = g_m / \sin k_{txm} t \tag{7f}$$

$$g_m = \left(\epsilon_m b k_{txm}\right)^{-1} \tag{7g}$$

$$\epsilon_m = 1, \quad \text{if } m = 0; \qquad 1/2, \quad \text{if } m \neq 0 \tag{7h}$$

$$X_{jqm} = \int_{h_j}^{h_j + d_j} \cos \frac{q\pi}{d_j}(y - h_j) \cos \frac{m\pi}{b} y \, dy \tag{8}$$

$$k_c^2 = k_{txm}^2 + (m\pi/b)^2 = k_{x1q}^2 + (q\pi/d_1)^2 = k_{x2q}^2 + (q\pi/d_2)^2. \tag{9}$$

The corresponding expressions for the TM modes are obtained by replacing cos by sin functions in (1) and (9), and the functions of the form $(x \sin x)^{-1}$ and $(\cot x)/x$ are replaced by $x/\sin x$ and $x \cot x$, respectively, in (6) and (7).

B. Solution of Eigenvector

To solve the eigenvector C, one particular component $C_l^{(j,k)}$ ($1 = 0, 1, \cdots, N$ for TE; $1, 2, \cdots, N$ for TM) is made aritrary. The other components of C are then expressed in terms of $C_l^{(j,k)}$ by solving the following set of matrix equations, which are easily obtained from (2):

$$\begin{aligned}
&[H_{1NN}]C_N^{(1,1)} + [H_{2NN}]C_N^{(1,2)} = -[H_{1N0}]C_l^{(1,1)} \\
&\cdot[H_{2NN}]C_N^{(1,1)} + [H_{3NN}]C_N^{(1,2)} + [H_{4NN}]C_N^{(2,1)} \\
&= -[H_{3N0}]C_l^{(1,2)} - [H_{4N0}]C_l^{(2,1)} \\
&\cdot[H_{5NN}]C_N^{(1,2)} + [H_{6NN}]C_N^{(2,1)} + [H_{7NN}]C_N^{(2,2)} \\
&= -[H_{5N0}]C_l^{(1,2)} - [H_{6N0}]C_l^{(2,1)} \\
&[H_{7NN}]C_N^{(2,1)} + [H_{8NN}]C_N^{(2,2)} = -[H_{8N0}]C_l^{(2,2)}.
\end{aligned} \tag{10}$$

In (10), H_{pNN} $(p = 1, 2, \cdots, 8)$ is the matrix derived from the submatrix H_p by deleting the lth row and lth column; H_{pN} is the column vector derived from the lth column of H_p by deleting the lth, and $C_N^{(j,k)}$ is the column vector derived from $C^{(j,k)}$ by deleting the lth row.

C. Gap Impedance

The dominant mode gap is impedance Z_g is defined as

$$Z_g = V_0^2 / 2P_0 \tag{11}$$

where P_0 is the power propagating in the guide, and V_0 is the gap voltage defined through

$$V_0 = \int_h^{h+d} e_y(x\,y)\, dy \qquad (x \text{ in gap}). \tag{12}$$

If the basis fields are made orthonormal, then, following Montgomery [1], Z_g can be expressed in terms of $C_0^{(j,k)}$ as given below. At $x = a_1 + \rho s_1$, $0 \leqslant \rho \leqslant 1$ for the first gap

$$Z_{g1} = Z_{g1(\infty)} \cdot \left[1 - (\lambda/\lambda_c)^2\right]^{-1/2} \tag{13a}$$

where $Z_{g1(\infty)}$ is the gap impedance at infinite frequency and is given by

$$Z_{g1(\infty)} = \frac{120\pi d_1^2}{\sin^2 k_c s_1} \cdot \left[C_0^{(1,1)} \sin k_c s_1 (1 - \rho) + C_0^{(1,2)} \sin k_c s_1 \rho\right]^2. \tag{13b}$$

For the second gap, that is, at $x = a_2 + \rho s_2$, Z_{g2} is obtained by replacing $C_0^{(1,1)}$, $C_0^{(1,2)}$, d_1, and s_1 by $C_0^{(2,1)}$, $C_0^{(2,2)}$, d_2, and s_2, respectively, in (13b).

TABLE I
NORMALIZED CUTOFF WAVELENGTHS FOR THE DOMINANT TE (λ_{c-}/a) AND THE FIRST HIGHER ORDER MODE (λ_{c+}/a) AND PERCENT BANDWIDTH (B) FOR $b/a = 0.5$, $d/b = 0.1$, $s/a = 0.125$ COMPUTED WITH $N = 5$, $M = 10$

h/b	Structure	t/a=.125			t/a=.250			t/a=.375			t/a=.500			t/a=.625			t/a=.750		
0.9	R	5.429	2.645	69.0	5.065	3.065	49.2	4.527	3.198	34.4	3.852	3.065	22.8	3.014	2.645	13.1	1.564	1.060	38.4
	I	6.244	2.315	91.8	5.340	2.301	59.2	4.534	3.113	39.1	3.393	3.030	25.0	3.029	2.631	14.1	1.565	1.048	39.6
0.7	R	4.992	2.230	74.6	4.596	2.706	51.8	4.082	2.338	36.0	3.449	2.706	24.1	2.648	2.230	15.0	1.560	1.026	41.2
	I	5.467	2.074	90.0	4.755	2.607	58.4	4.145	2.739	39.1	3.474	2.684	25.6	2.657	2.270	15.7	1.560	1.021	41.8
0.5	R	4.914	2.102	80.2	4.505	2.567	54.8	3.972	2.707	37.9	3.320	2.567	25.6	2.494	2.102	17.2	1.559	0.999	43.8
	I	4.949	2.086	81.4	4.514	2.561	55.2	3.975	2.705	38.0	3.322	2.566	25.7	2.494	2.101	17.1	1.559	0.999	43.8
0.45	R,I	4.921	2.097	80.5	4.511	2.566	55.0	3.975	2.707	38.0	3.321	2.566	25.7	2.491	2.097	17.2	1.559	0.997	43.9

Structure: R—REGULAR, I—INVERTED

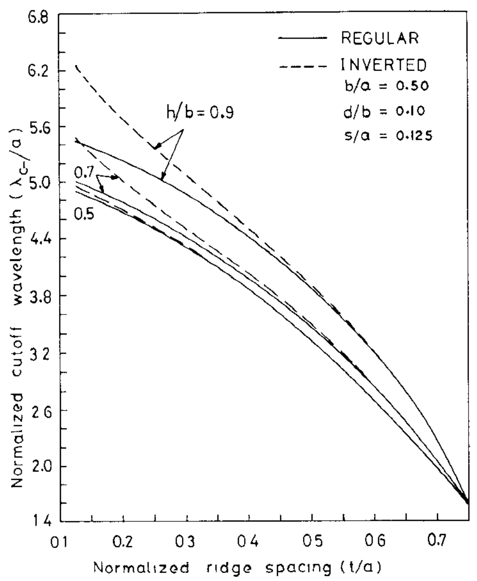

Fig. 2. Variation of normalized cutoff wavelength (λ_c/a) of dominant TE mode with ridge spacing (t/a) in regular and inverted guides

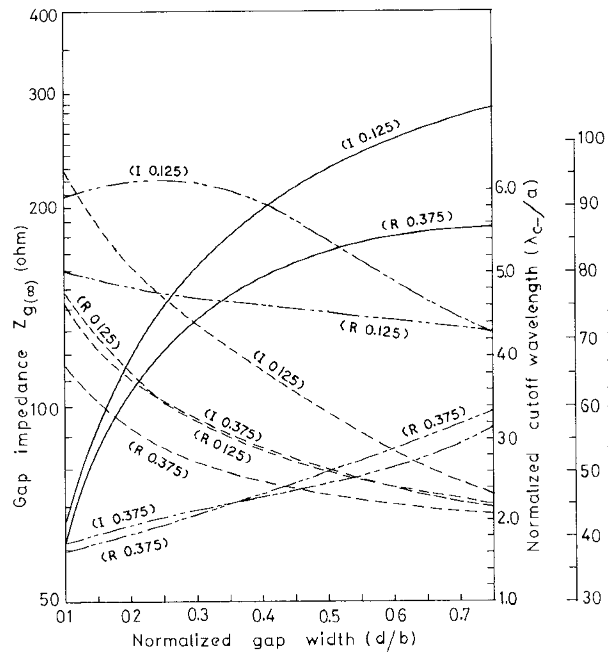

Fig 3. Variation of percent bandwidth (B), normalized cutoff wavelength (λ_{c-}/a) and dominant TE-mode gap impedance $Z_{g(\infty)}$ with normalized gap with (d/b). $b/a = 0.5$, $s/a = 0.125$.——$Z_{g(\infty)}$. ----(λ_{c-}/a).-·-·- B. The parameters indicated in parentheses refer to the type of structure, and the value of ridge spacing t/a.

IV. NUMERICAL RESULTS

A. Cutoff Wavelength and Bandwidth

The matrix eigenvalue equation was solved [2] to determine the normalized cutoff wavelengths (λ_{c-}/a) of the dominant TE mode and (λ_{c+}/a) of the next higher order mode for both the "regular" and "inverted" structures. For a fixed ridge width s/a and a fixed gap width d/b, the gap location h/b and the ridge spacing t/a were varied, including the case when the ridges are in contact with the sidewalls. The percent bandwidth B was calculated from

$$B = 200 \times (\lambda_{c-} - \lambda_{c+})/(\lambda_{c-} + \lambda_{c+}). \tag{14}$$

The results of computation are given in Table I. In all the cases tabulated, the first higher order mode is TE. The variation of (λ_{c-}/a) is also shown graphically in Fig. 2. The computations were carried out with six terms in the aperture field expansion ($N = 5$) and 11 terms for the trough region field ($M = 10$). The relative convergence behavior is similar to that reported in [2] and the values of λ_c/a obtained with $N = 10$ differ from those in Table I by less than 1 percent. We observe that the inverted structure always shows better bandwidth than that of its regular counterpart. The improvement is most significant when the ridge spacing is small ($t/a < 0.5$) because of increased capacitive loading of the guide. For the ridge parameters chosen, the bandwidth is the largest (92 percent) for an inverted structure with two single ridges antisymmetrically placed at $t/a = 0.125$. This may be compared with the bandwidth of 69 percent for the corresponding regular structure with identical parameters. In regular configuration, the maximum bandwidth available is 81 percent when the gaps are centered or each ridge is a symmetrical double ridge. Some additional information is given in Fig. 3, where, for a fixed ridge width (s/a) and two values of ridge spacing (t/a), the variations of the normalized cutoff wavelength (λ_{c-}/a) and the bandwidth B with gap width (d/b) are shown for a regular structure with two symmetrical double ridges and an inverted structure with two single ridges.

B. Eigenvector and Gap Impedance

To solve the eigenvector $C_N^{(j,k)}$ in (10) was expressed in terms of $C_I^{(j,k)}$ and inverted submatrices after further manipulations. However, these expressions were not useful for numerical computation, as the diagonal elements of diagonal matrices H_2 and H_7 are all zero after the first few leading elements, which cause failure in numerical inversion. The actual computation of the eigenvector components was carried out by a Gauss–Siedel iteration of (10) with $C_I^{(j,k)}$ made arbitrary successively (usually unity prior to normalization of the eigenvector). The matrix $[H]$ being diagonally dominant, the convergence was excellent.

The variation of $Z_{g(\infty)}$ at the gap center $(\rho = 0.5)$ with (d/b) for the dominant TE mode in an inverted structure with two single ridges is shown in Fig. 3. For a given set of parameters (t/a), (s/a), and (d/b), this impedance is found to be almost independent of gap height (h/b), that is, almost identical for the regular and inverted structures. For given (s/a) and (d/b), however, it varies considerably with ridge spacing (t/a) in either case. The impedance curves in Fig. 3 also closely depict those for the regular structure with two symmetrical double ridges of identical parameters, with the difference of impedance in the two cases being less than 0.5 percent.

V. Ridged Waveguide Varactor-Tuned Gunn Oscillator

In this section, some preliminary experimental results obtained with two empirically-designed XN-band varactor-tuned Gunn oscillators in fixed-length ridged-waveguide resonators are presented. The device mounts were not optimized and the choice of ridge parameters was guided purely by mechanical considerations. Fig. 4(a) shows the schematic diagrams of the device mount in an inverted ridged WR-137 waveguide resonator (referred to as oscillator A). For the oscillator with a resonator in regular configuration (referred to as oscillator B), all the dimensions except the ridge spacing were identical. The performance of these two oscillators is shown in Fig. 4(b), where the measured shift in oscillation frequency and the output power are plotted as functions of varactor bias.

VI. Conclusions

The Ritz–Galerkin technique has been applied to determine the eigenvalues of a rectangular waveguide with two double ridges located arbitrarily. When two identical, asymmetric double ridges are placed symmetrically, considerable improvement in bandwidth occurs if one ridge is inverted with respect to the other. The best result is obtained when two single ridges are closely spaced in an inverted configuration. The eigenvector has been obtained by iteration of the matrix equations and has been used in determining the gap impedance. Finally, the same preliminary results obtained with two XN-band varactor-tuned Gunn oscillators in regular and inverted ridged resonators have been presented. Though there is scope for optimization of the device mounts, the results are certainly promising for such applications of the ridged waveguides.

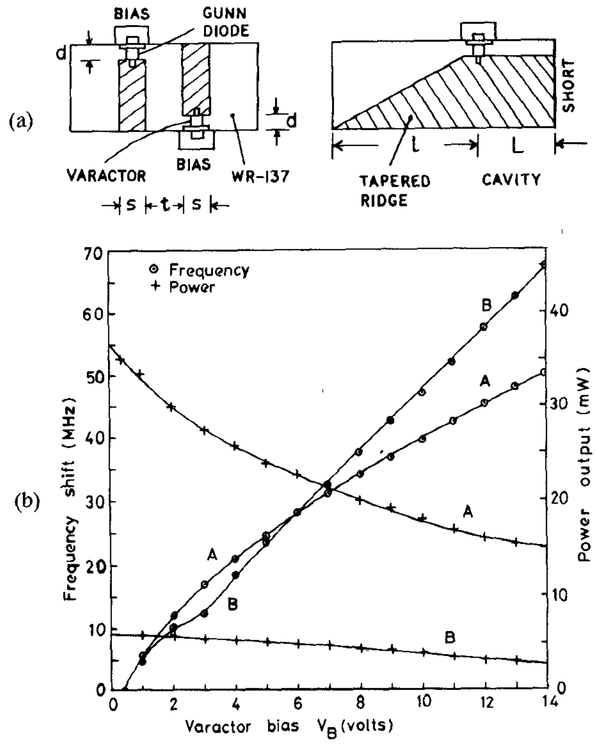

Fig. 4. (a) Varactor-tuned Gunn oscillator in inverted waveguide resonator (waveguide: WR-137). $s = 0.25$ in, $d = 0.1$ in, $l = 1.75$ in, $L = 0.875$ in $t =$ 0.157 in (OSC.A); 0.295 in (OSC.B). (b) Variation of frequency shift and power output with varactor bias (V_B). Frequency of oscillation at $V_B = 0.4$ V: 7.495 GHz (OSC.A); 7.20 GHz (OSC.B). Varactor diode: AEI DC 4201B, Gunn diode: MA 49151 (OSC.A); MA 49156 (OSC.B).

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Mutual Impedance Computation Between Microstrip Antennas

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Abstract—**A moment-method solution for the mutual coupling between rectangular microstrip antennas is presented. The grounded dielectric slab is accounted for exactly in the analysis.**

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